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ANALYSIS OF OPEN DIELECTRIC WAVEGUIDES USING MODE-MATCHING TECH--ETC(U)
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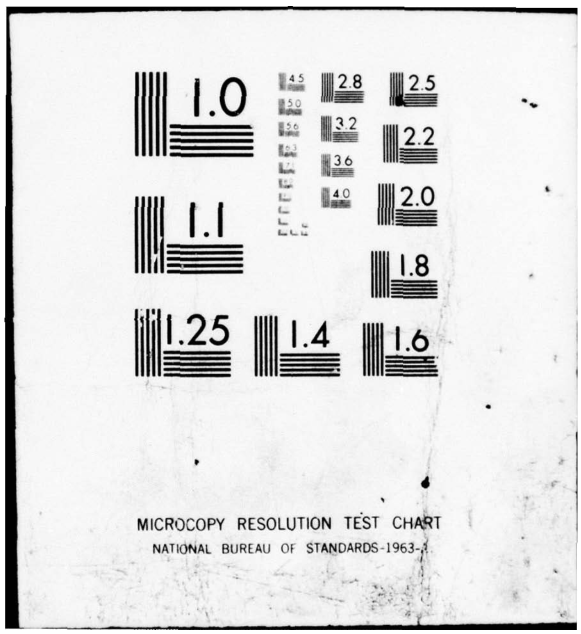
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ANALYSIS OF OPEN DIELECTRIC WAVEGUIDES USING MODE-MATCHING TECHNIQUE AND VARIATIONAL METHODS

Interim Technical Report

R. Mittra
Y. Hou
V. Jamnejad

March 1979

U. S. Army Research Office
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Electromagnetics Laboratory
Department of Electrical Engineering
University of Illinois
Urbana, IL 61801

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1. INTRODUCTION

Recent interest in the 30-300 GHz range, which has remained relatively unexplored hitherto, has led to the investigation of low-loss, low-cost dielectric waveguide designs ([1]-[4]) suitable for integrated circuit applications in this frequency range.

In order to develop reliable designs for uniform dielectric guides, as well as for active and passive components constructed from these waveguides, it is extremely important to have the capability of theoretically predicting the performance of these circuit elements and transmission media.

A search through the literature on optical and quasi-optical dielectric waveguides reveals, however, that the progress in this direction has been rather limited and the most commonly employed approach appears to be based on what is called the "effective dielectric constant" method ([1]-[4]). An alternate approach, called the "effective permeability" method, has also been developed [5]; however, both of these techniques are based on certain approximations that are neither easily justified nor always satisfied. Furthermore, they do not provide information on complete field distributions. Recently, an exact formulation of the problem for dielectric image guides has been developed which is based on the expansion of the field in each subregion of the guide cross-section into a complete set of functions, and the consequent matching at the boundaries [6]. The numerical results obtained from this method seem to be in good agreement with the experimental results.

In this paper, a rather similar approach based on the mode-matching technique [7] is used for a more complete analysis of the open, planar, dielectric waveguide problem, specifically, an homogeneous inverted strip guide (HIS). The method is quite general, and is useful even at optical frequencies. In addition, a single-mode approximation of the full-wave method presented here is found to be equivalent to employing the "effective dielectric constant" approach.

The propagation constant obtained from the mode-matching technique is further improved, by employing variational expressions which are modified for the present analysis.

2. MODE MATCHING

We consider the waveguide geometry in Fig. 1, which shows the cross-section of the homogeneous inverted strip guide (HIS). For simplicity of analysis, and in order to define a proper eigenvalue problem, a perfect electric conductor is placed parallel to, and at a large distance from the ground plane. The distance b is chosen large enough to make the influence of the conducting plane on the guide properties negligible. The guided modes in this structure will generally have all the components of E and H fields. Because of the symmetry of the structure with respect to the $x = 0$ plane (Fig. 1), symmetric (even) and antisymmetric (odd) modes can propagate in the guide. If E_y is even (or H_y is odd), we can insert a magnetic wall at the $x = 0$ plane, without affecting the field distribution. Similarly, if E_y is odd (or H_y is even), we can introduce an electric wall at the $x = o$ plane. In either case, we need to consider only half of the guide cross-section. Here, we consider only the E_y-symmetric case (magnetic wall at $x = 0$) and the right half of the guide cross-section. The antisymmetric case can be similarly studied.

As shown in Fig. 1, we divide the region under consideration (right half of the cross-section) into two subregions. The field in each region can be expanded in terms of its eigenfunctions. Next, we match the fields at the interface $x = w$ and solve for the prapagation constant, as well as the field distribution.

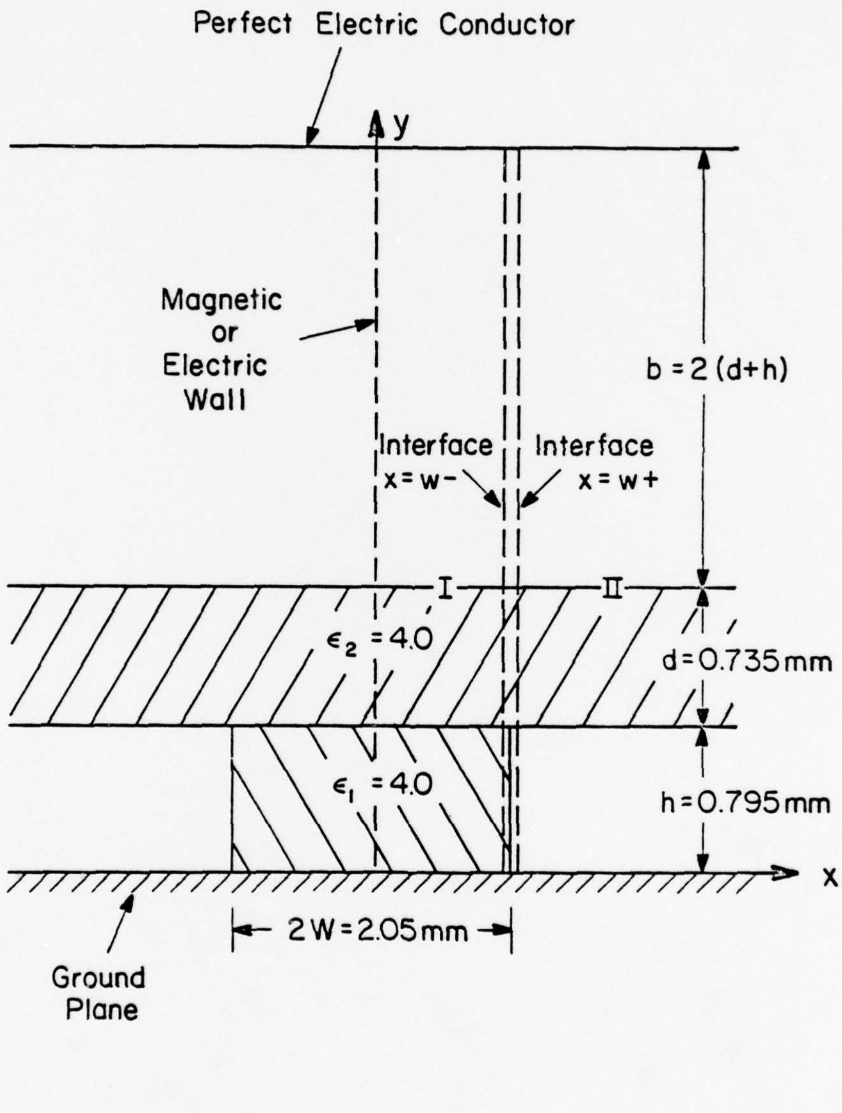

Figure 1. Cross-section of homogeneous inverted strip guide.

2.1. Modes in Regions I and II

The modes can be classified as TM or TE with respect to the y-direction [8]. The fields can be expressed in terms of the scalar potential functions $f^e(x,y)$ and $f^h(x,y)$ as follows.

TM Modes:

$$E_{xm} = \frac{1}{\varepsilon(y)} \frac{\partial^2 f_m^e(x,y)}{\partial x \; \partial y}$$

$$E_{ym} = \frac{1}{\varepsilon(y)} \left(k_z^2 - \frac{\partial^2}{\partial x^2}\right) f_m^e(x,y)$$

$$E_{zm} = \frac{-jk_z}{\varepsilon(y)} \frac{\partial f_m^e(x,y)}{\partial y}$$

$$H_{xm} = -\omega k_z \, \varepsilon_0 \, f_m^e(x,y) \tag{1}$$

$$H_{ym} = 0$$

$$H_{zm} = j \, \omega \, \varepsilon_0 \frac{\partial f_m^e(x,y)}{\partial x}$$

The potential function of the m-th TM mode is separable and can be written as:

$$f_m^e\,(x,y) = \phi_m^e\,(y)\,\cos(k_{xm}x) \tag{2}$$

and

$$\phi_m(y) = \begin{cases} \cos\,(k_{y1m}\,y) & \text{for} \quad 0 < y < h \\ \alpha_m^e \cos\,[k_{y2m}\,(y-h)] + \beta_m^e \sin[k_{y2m}\,(y-h)] & \text{for } h \leq y \leq (h+d) \\ \gamma_m^e \cos\,[k_{y3m}\,(h+b+d-y)] & \text{for } (h+d) \leq y \leq (h+d+b) \end{cases} \tag{3}$$

in which

$$\alpha_m^e = \cos\,(k_{y1m}\,h)$$

$$\beta_m^e = -\frac{\varepsilon_2}{\varepsilon_1}\,\frac{k_{y1m}}{k_{y2m}}\,\sin(k_{y1m}\,h)$$

$$\gamma_m^e = [\cos\,(k_{y1m}\,h)\,\cos\,(k_{y2m}\,d) - \frac{\varepsilon_2}{\varepsilon_1}\,\frac{k_{y1m}}{k_{y2m}}\,\sin\,(k_{y1m}\,h)\,\sin\,(k_{y2m}\,d)]\,/\,\cos\,(k_{y3m}\,b)$$

$$k_{y1m} = (\varepsilon_1\,k_0^2 - k_{xm}^2 - k_z^2)^{1/2}$$

$$k_{y2m} = (\varepsilon_2 k_0^2 - k_{xm}^2 - k_z^2)^{1/2}$$

$$k_{y3m} = (\varepsilon_3\,k_0^2 - k_{xm}^2 - k_z^2)^{1/2}$$

$$k_0^2 = \omega^2\,\mu_0\varepsilon_0\,.$$

In Region I, ε_1, ε_2, and ε_3 are relative dielectric constants of the dielectric strip, guiding layer, and air, respectively.

The eigenvalue equation for $k_x^2 + k_z^2$ is

$$(k_{y1}/\varepsilon_1) \tan (k_{y1} h) + (k_{y2}/\varepsilon_2) \tan (k_{y2} d) + (k_{y3}/\varepsilon_3) \tan (k_{y3} b) =$$

$$[(k_{y1}/\varepsilon_1) (k_{y3}/\varepsilon_3) / (k_{y2}/\varepsilon_2)] \tan (k_{y1} h) \tan (k_{y2} d) \tan (k_{y3} b) \tag{4}$$

<u>TE Modes</u>:

$$E_{xm} = \omega\mu_0 \; k_z \; f_m^h (x,y)$$

$$E_{ym} = 0$$

$$E_{zm} = -j\omega \, \mu_0 \, \frac{\partial f_m^h(x,y)}{\partial x}$$

$$H_{xm} = \frac{\partial^2 \, f_m^h(x,y)}{\partial y \, \partial x} \tag{5}$$

$$H_{ym} = (k_z^2 - \frac{\partial^2}{\partial x^2} \quad f_m^h(x,y)$$

$$H_{zm} = -j \, k_z \frac{\partial f_m^h \, (x,y)}{\partial y}$$

The potential function of the m-th TE mode, $f_m^h(x,y)$, is also separable and can be written as:

$$f_m^h \, (x,y) = \phi_m^h(y) \, \sin(k'_{xm} x) \tag{6}$$

and

$$\phi_m^h(y) = \begin{cases} \sin(k'_{y1m}\, y) & \text{for } 0 \le y \le h \\ \alpha_m^h \cos\,[k'_{y2m}\,(y-h)] + \beta_m^h \sin[k'_{y2m}(y-h)] & \text{for } h \le y \le h+d \\ \alpha_m^h \sin[k'_{y_3m}\,(h+d+b-y)] & \text{for } (h+d) \le y \le (h+d+b) \end{cases} \tag{7}$$

in which

$$\alpha_m^h = \sin(k'_{y1m}\, h)$$

$$\beta_m^h = \frac{k'_{y1m}}{k'_{y2m}} \cos(k'_{y1m} h)$$

$$\alpha_m^h = [\sin(k'_{y1m} h)\cos(k'_{y2m} d) + \frac{k'_{y1m}}{k'_{y2m}} \cos(k'_{y1m} h)$$

$$\sin(k'_{y2m} d)]/\sin(k'_{y3m} b)$$

$$k'_{y1m} = (\varepsilon_1\, k^2 - k'^2_{xm} - k_z^2)^{1/2}$$

$$k'_{y2m} = (\varepsilon_2\, k^2 - k'^2_{xm} - k_z^2)^{1/2}$$

$$k'_{y3m} = (\varepsilon_3\, k^2 - k'^2_{xm} - k_z^2)^{1/2}$$

The eigenvalue equation for $k'^2_x + k^2_z$ is

$$(1/k'_{y1})\ \tan(k'_{y1}h) + (1/k'_{y2})\ \tan(k'_{y2}d) + (1/k'_{y3})\ \tan(k'_{y3}b) =$$

$$[k'_{y2}/(k'_{y1}\ k'_{y3})]\ \tan(k'_{y1}h)\ \tan(k'_{y2}d)\ \tan(k'_{y3}b)$$

This completes the discussion of the modes in Region I. The modes in Region II can be obtained from those in Region I by simply setting $\varepsilon_1 = 1$, and assuming an exponentially decaying wave in the x-direction.

2.2. Fields Expansion and Matching

The fields in each of the two Regions I and II are first expanded in terms of the TE and TM mode functions. The expansion coefficients and the propagation constant (k_z) are then obtained by matching the tangential components of the fields at the interface plane x = w.

Fields Expansion in Region I

$$E_y = \sum_{m=1}^{M} (k_{xm}^2 + k_z^2)\ \frac{\cos(k_{xm}x)}{\cos(k_{xm}w)}\ \frac{\phi_m^e(y)}{\varepsilon(y)}\ A_m$$

$$H_y = \sum_{m=1}^{M'} (k'^2_{xm} + k_z^2)\ \frac{\sin(k'_{xm}x)}{\sin(k'_{xm}w)}\ \phi_m^h(y)\ B_m$$

$$E_z = \sum_{m=1}^{M} (-jk_z)\ \frac{\cos(k_{xm}x)}{\cos(k_{xm}w)}\ \frac{1}{\varepsilon(y)}\ \frac{\partial\phi_m^e(y)}{\partial y}\ A_m$$

$$+ \sum_{m=1}^{M'} (-j\omega\mu_0 k'_{xm})\ \frac{\cos(k'_{xm}x)}{\sin(k'_{xm}w)}\ \phi_m^h(y)\ B_m$$

$$H_z = \sum_{m=1}^{M} (-j\omega\varepsilon_0\ k_{xm})\ \frac{\sin(k_{xm}x)}{\cos(k_{xm}w)}\ \phi_m^e(y)\ A_m$$

$$+\sum_{m=1}^{M'} (-jk_z) \frac{\sin(k'_{xm}x)}{\sin(k'_{xm}w)} \frac{\partial f_m^h(y)}{\partial y} B_m$$

$$E_x = \sum_{m=1}^{M} (-k_{xm}) \frac{\sin(k_{xm}x)}{\cos(k_{xm}w)} \frac{1}{\varepsilon(y)} \frac{\partial \phi_m^e(y)}{\partial y} A_m$$

$$+\sum_{m=1}^{M'} (\omega\mu_0 k_z) \frac{\sin(k'_{xm}x)}{\sin(k'_{xm}w)} \phi_m^h(y) B_m$$

$$H_x = \sum_{m=1}^{M} (-\omega\varepsilon_0 k_z) \frac{\cos(k_{xm}x)}{\cos(k_{xm}w)} \phi_m^e(y) A_m$$

$$+\sum_{m=1}^{M} (k'_{xm}) \frac{\cos(k'_{xm}x)}{\sin(k'_{xm}w)} \frac{\partial \phi_m^h(y)}{\partial y} B_m \tag{9}$$

A_m's, B_m's and k_z in Eqs. (9) are constants to be determined.

Fields Expansion in Region II

$$E_y = \sum_{m=1}^{N} (\bar{k}_{xm}^2 + k_z^2)\, e^{-j\bar{k}_{xm}(x-w)} \frac{\bar{\phi}_m^e(y)}{\bar{\varepsilon}(y)} C_m$$

$$H_y = \sum_{m=1}^{N'} (\bar{k}'^2_{xm} + k_z^2)\, e^{-j\bar{k}'_{xm}(x-w)} \bar{\phi}_m^h(y) D_m$$

$$E_z = \sum_{m=1}^{N} (-jk_z)\; e^{-j\bar{k}_{xm}(x-w)} \frac{1}{\bar{\varepsilon}(y)} \frac{\partial \bar{\phi}_m^e(y)}{\partial y}\; C_m$$

$$+ \sum_{m=1}^{N'} (-\omega\mu_0 \bar{k}'_{xm})\; e^{-j\bar{k}'_{xm}(x-w)}\; \bar{\phi}_m^h(y)\; D_m$$

$$H_z = \sum_{m=1}^{N} (\omega\varepsilon_0\, \bar{k}_{xm})\; e^{-j\, \bar{k}_{xm}(x-w)}\; \bar{\phi}_m^e(y)\; C_m$$

$$+ \sum_{m=1}^{N'} (-jk_z)\; e^{-j\bar{k}'_{xm}(x-w)} \frac{\partial \bar{\phi}_m^h(y)}{\partial y}\; D_m$$

$$E_x = \sum_{m=1}^{N} (-j\bar{k}_{xm})\; e^{-j\bar{k}_{xm}(x-w)} \frac{1}{\bar{\varepsilon}(y)} \frac{\partial \bar{\phi}_m^e(y)}{\partial y}\; C_m$$

$$+ \sum_{m=1}^{N'} (\omega\mu_0\, k_z)\; e^{-j\bar{k}'_{xm}(x-w)}\; \bar{\phi}_m^h(y)\; D_m$$

$$H_x = \sum_{m=1}^{N} (-\omega\varepsilon_0\, k_z)\; e^{-j\bar{k}_{xm}(x-w)}\; \bar{\phi}_m^e(y)\; C_m$$

$$+ \sum_{m=1}^{N'} (-j\bar{k}'_{xm})\; e^{-j\bar{k}'_{xm}(x-w)} \frac{\partial \bar{\phi}_m^h(y)}{\partial y}\; D_m \qquad (10)$$

In Eqs. (10) the barred characters are used to distinguish the values in Region II from those in Region I, and C_m's and D_m's are constants to be determined.

Matching of Tangential Field Components at $x = w$

Required continuity of the field components E_y, H_y, E_z, and H_z, as given in Eqs. (9) and (10), across the plane $x = w$, leads to the following equations:

$$\sum_{m=1}^{M} (k_{xm}^2 + k_z^2) \frac{\phi_m^e(y)}{\varepsilon(y)} A_m - \sum_{m=1}^{N} (\bar{k}_{xm}^2 + k_z^2) \frac{\bar{\phi}_m^e(y)}{\bar{\varepsilon}(y)} C_m = 0 \tag{11}$$

$$\sum_{m=1}^{M'} (k'^2_{xm} + k_z^2) \phi_m^h(y) B_m - \sum_{m=1}^{N'} (\bar{k}'^2_{xm} + k_z^2) \bar{\phi}_m^h(y) D_m = 0 \tag{12}$$

$$\sum_{m=1}^{M} \frac{1}{\varepsilon(y)} \frac{\partial \phi_m^e(y)}{\partial y} A_m + \sum_{m=1}^{M'} \frac{\omega\mu_0 k'_{xm}}{k_z} \cot(k'_{xm} w) \phi_m^h(y) B_m$$

$$-\sum_{m=1}^{N} \frac{1}{\bar{\varepsilon}(y)} \frac{\partial \bar{\phi}_m^e(y)}{\partial y} C_m - \sum_{m=1}^{N'} \frac{-j\omega\mu_0 \bar{k}'_{xm}}{k_z} \bar{\phi}_m^h(y) D_m = 0 \tag{13}$$

$$\sum_{m=1}^{M} \frac{\omega\varepsilon_0 k_{xm}}{k_z} \tan(k_{xm} w) \phi_m^e(y) A_m + \sum_{m=1}^{M'} \frac{\partial f_m^h(y)}{\partial y} B_m$$

$$-\sum_{m=1}^{N} \frac{j\omega\varepsilon_o \bar{k}_{xm}}{k_z} \bar{\phi}_m^e(y) C_m - \sum_{m=1}^{N'} \frac{\partial \bar{\phi}_m^h(y)}{\partial y} D_m = 0 \tag{14}$$

Equations (11) - (14) yield an exact solution for the fields and propagation constants if M, N, M' and N are infinite. However, in practice, one must limit these to finite numbers. As a consequence of this approximation, the field matching at the interface is not perfect and there is a residual discontinuity of the tangential fields as one traverses the interface. We will address this problem a little later when we derive the variational expressions for the propagation constants.

The following orthogonality relations can be shown to hold and are utilized in solving Equations (11) - (14).

$$\int_0^{h+d+b} \frac{1}{\varepsilon(y)} \; \phi_m^e(y) \; \phi_n^e(y) \; dy = 0$$

$$\int_0^{h+d+b} \phi_m^h(y) \; \phi_n^h(y) \; dy = 0$$

$$\int_0^{h+d+b} \frac{1}{\bar{\varepsilon}(y)} \; \bar{\phi}_m^e(y) \; \bar{\phi}_n^e(y) \; dy = 0 \qquad \text{for } m \neq n \qquad (15)$$

$$\int_0^{h+d+b} \bar{\phi}_m^h(y) \; \bar{\phi}_n^h(y) \; dy = 0$$

Notice that no cross-orthogonality relations exist between the different potential functions. Any two or all four of the relations in (15) can be used in setting up a system of linear homogeneous equations. Next, we assume an equal number of TM modes in the two Regions I and II (i.e. $M = N$), and also an equal number of TE modes (i.e. $M'=N'$). We then multiply Equation (11) by $\phi_n^e(y)$, Equation (14) by $\phi_n^e(y)/\varepsilon(y)$, for $n=1, \ldots, M$, and Equations (12) and (13) by $\phi_n^h(y)$, for $n=1, \ldots, M'$. Finally, we integrate the products over the interval $0 \leq y \leq (h+d+b)$. Using the first two orthogonality relations in (15), we obtain a system of $2(M+M')$ linear homogeneous equations for the unknowns A_n, C_n, for $n=1, \ldots, M$, and B_n, D_n, for $n=1, \ldots, M'$. The next step is to solve for A_n's and B_n's from the set of equations generated from

(11) and (12) and then insert the results in the set of equations generated from (13) and (14). This procedure yields a reduced system of (M+M') homogeneous equations for the unknowns C_n, with n=1, ...M, and the unknowns D_n, with n=1, ...M'. The zeros of the determinant of this system of equations are the desired eigenvalues or the propagation constants k_z. Each k_z relates to a particular guide mode for which C_n's, D_n's, A_n's and B_n's are found within a constant multiplicative factor.

3. REDUCTION TO EFFECTIVE ε CONCEPT

It can be shown that solutions based on "effective dielectric constant" concepts ([3], [4]) are essentially one-term approximations of the present analysis. This is illustrated by setting M=N=1 and M'=N'=0. Then following the steps outlined in Section 2, the following eigenvalue equation for k_z is obtained.

$$k_{x1} \tan(k_{x1} w) = j\, \bar{k}_{x1} \left[\frac{(k_{x1}^2 + k_z^2)}{(\bar{k}_{x1}^2 + k_z^2)} \frac{\int_0^{h+d+b} (\phi_1^e(y)\, \bar{\Phi}_1(y)/\varepsilon(y))\, dy}{\int_0^{h+d+b} (\phi_1^e(y)\, \bar{\Phi}_1(y)/\bar{\varepsilon}(y))\, dy} \right] \tag{16}$$

The quantity in the square brackets in Equation (16) can be assumed to be approximately one. Then a comparison of Equation (16) with the eigenvalue obtained from the "effective permittivity" approach (e.g., Equation (9) in [4]) shows them to be identical.

Therefore, the method presented in the present work is a generalization of the existing approaches that makes is possible to derive higher-order approximations in a systematic manner.

4. VARIATIONAL IMPROVEMENTS

The mode-matching results for the propagation constants can be further improved via the use of the variational techniques. The field distribution is, in general, discontinuous across the interface of the Regions I and II where the matching takes place. This, as has already been mentioned in Section II, is due to the approximation introduced by the use of a finite number of TM and TE modes in the process of matching the field across the interface $x = w$. The presence of such discontinuities requires that the conventional variational formulas for obtaining the propagation constant from the approximate fields distributions ([8], [9]) be suitably modified. Although three different formulations (E-field, H-field, and mixed-field) of the variational problem are possible, only the mixed-field formulation is considered here.

4.1. Mixed-Field Formulation

Following Harrington's formulation of the problem [8], we define the waves traveling in the +z direction in the guide as:

$$\vec{E}^{+} = \vec{E}^{+}(x,y)\, e^{-jk_z z} = (\vec{E}_t + \hat{z} E_z)\, e^{-jk_z z}$$
$$\vec{H}^{+} = \vec{H}^{+}(x,y)\, e^{-jk_z z} = (\vec{H}_t + \hat{z} H_z)\, e^{-jk_z z} \qquad (17)$$

It can be shown that for any traveling wave solution in the +z direction, there exists a corresponding traveling wave solution in the -z direction given by:

$$\vec{E}^{-} = \vec{E}^{-}(x,y)\, e^{+jk_z z} = (\vec{E}_t - \hat{z}\, E_z)\, e^{+jk_z z}$$

$$\vec{H}^{+} = \vec{H}^{-}(x,y)\, e^{+jk_z z} = (-\vec{H}_t + \hat{z}\, H_z)\, e^{+jk_z z} \tag{18}$$

Then, the mixed-field stationary formula for k_z is found to be (see Appendix I)

$$k_z = \frac{\mathcal{M} + \iint (\omega\varepsilon\, \vec{E}^{+}\cdot\vec{E}^{-} - \omega\mu\vec{H}^{+}\cdot\vec{H}^{-} + j\vec{H}^{-}\cdot\nabla\times\vec{E}^{+} + j\vec{E}^{+}\cdot\nabla\times\vec{H}^{-})\, ds}{2\iint \vec{E}_t \times \vec{H}_t \cdot \hat{z}\; ds} \tag{19}$$

in which

$$\mathcal{M} = j \int_c \hat{n} \cdot (\vec{E}_p^{+} \times \vec{H}_m^{-} - \vec{E}_m^{+} \times \vec{H}_p^{-})\, d\ell \tag{20}$$

The double integrations in (19) are over the cross-sectional surface of the guide and the line integration in (20) is performed along the interface where the field is discontinuous. The subscripts "p" and "m" in (20) indicate opposite sides of the discontinuity surface while $\hat{n}$ is the unit normal to this surface directed from "m" to "p." The expression (19) is valid when the trial fields satisfy the boundary condition on the walls of the guide, although they may be discontinuous across some interface c in the cross-section of the guide. If no such discontinuity exists, $\mathcal{M}$, as given in (20), becomes zero and the variational formula (19) reduces to a simpler one given in [8].

For the geometry in Figure 1, $\hat{n} = \hat{x}$ and subscripts "p" and "m", indicate the fields at $x = w^{+}$ and $x = w^{-}$, respectively.

5. NUMERICAL RESULTS

5.1. Results of Mode-Matching Calculations

In this section, we present some representative results based on the analytical procedures described in Section 2. All of the results pertain to the homogeneous inverted strip guide of Figure 1, and are computed for 79.4 GHz to coincide with the experimental measurements.

Table I shows the convergence of the results for the propagation constant with the increase in the number of TE^y and TM^y modes retained in the mode-matching calculations. The results obtained from the "effective dielectric constant" approach, which is equivalent to a single-mode approximation,and the measured values of k_z are also included in the table. It should be pointed out that although the propagation constant for the dominant mode as computed from the "effective dielectric constant" method is quite accurate, neither the results for the field distribution of this mode, nor the propagation constants of the higher-order modes are comparable in accuracy to those derived from the mode-matching procedure.

Figures 2-5 compare the tangential field components of the E^y_{11} guide mode at $x = w^-$ and $x = w^+$. The H_y and H_z fields are extremely well-matched when 7 TE and 7 TM modes are used for the field expansion in the two regions, $0 \leq x \leq w^-$ and $n \geq w^+$. However, the E field match improves only slightly as the number of modes used is increased. The matching of E_y is more difficult since it is a continuous function of y at $x = w^-$ and $y = h$, while a discontinuous function of y at $x = w^+$ and $y = h$. However, except for this difficulty at $x = w$, the matching process for the E_y field converged rapidly at other values of x as is evident from Figures 6 and 7. Figure 6 shows the distribution of D_y at the $x = 0$

TABLE I.

PROPAGATION CONSTANTS OF THE GUIDED MODES IN HOMOGENEOUS INVERTED STRIP GUIDE (SHOWN IN FIGURE1) AT FREQUENCY 79.4 GHz

A. E^y_{11}

	1 TE 1 TM	3 TE 3 TM	5 TE 5 TM	7 TE 7 TM	Effective ε	Experiment
k_z (mm^{-1})	2.97181	2.98916	2.98721	2.98726	2.9906	3.0

B. H^y_{11}

	1 TE 1 TM	3 TE 3 TM	5 TE 5 TM	7 TE 7 TM	Effective μ
k_z (mm^{-1})	2.73411	2.62101	2.72954	2.72033	2.75947

C. E^y_{21}

	1 TE 1 TM	3 TE 3 TM	5 TE 5 TM	7 TE 7 TM	Effective ε
k_z (mm^{-1})	2.36457	2.38713	2.39104	2.39080	2.40701

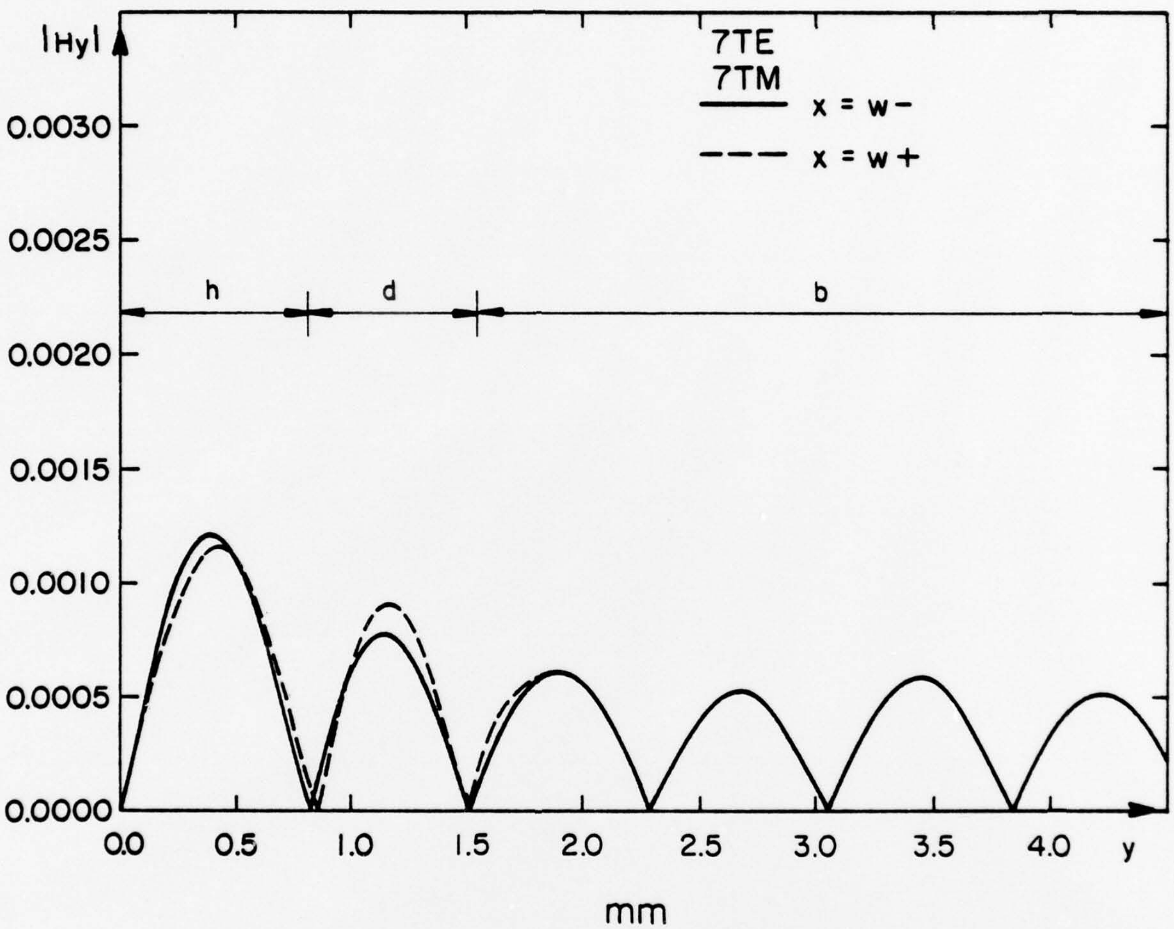

Figure 2. Plot of $|H_y|$ field of the E_{11}^y mode at the interface for $x = w^-$ and $x = w^+$.

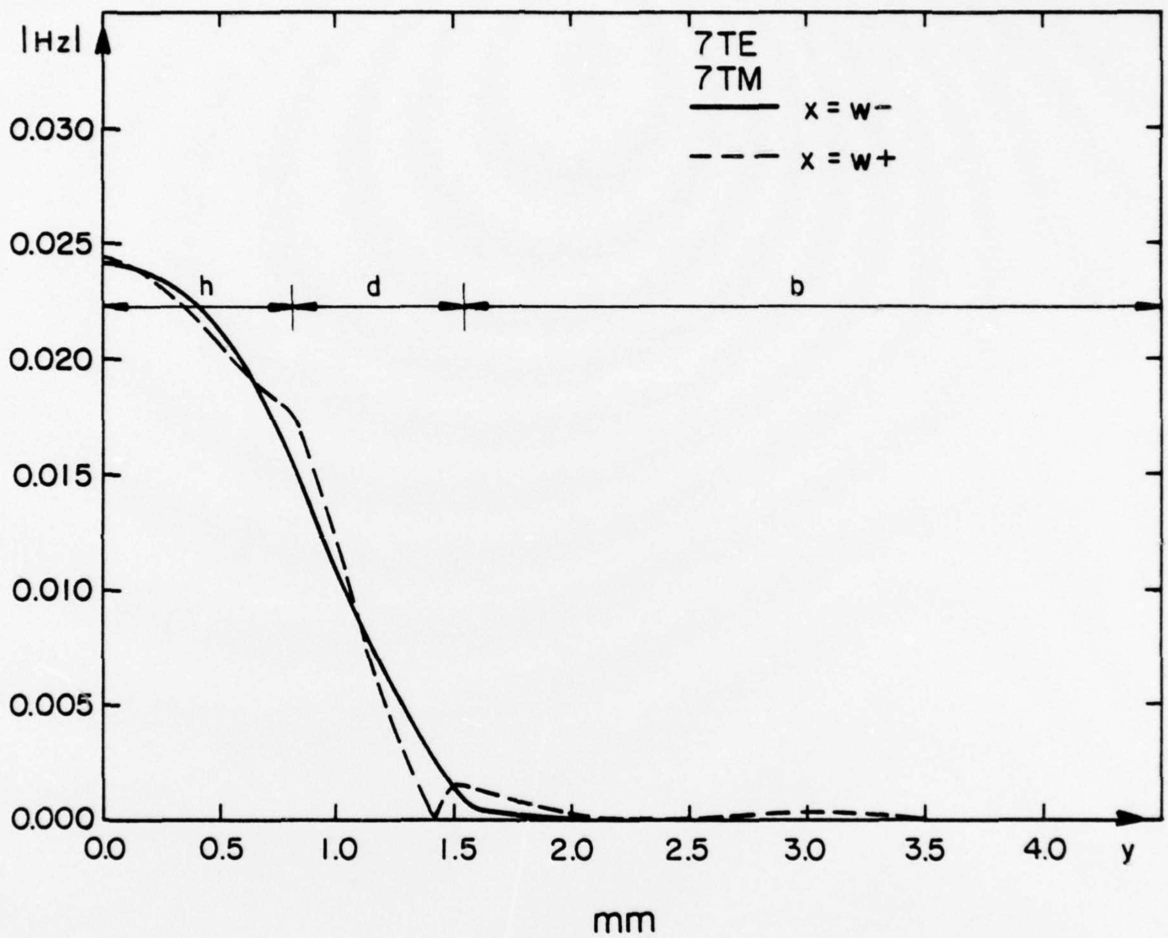

Figure 3. Plot of $|H_z|$ field of the E_{11}^y mode at the interface for $x = w^-$ and $x = w^+$.

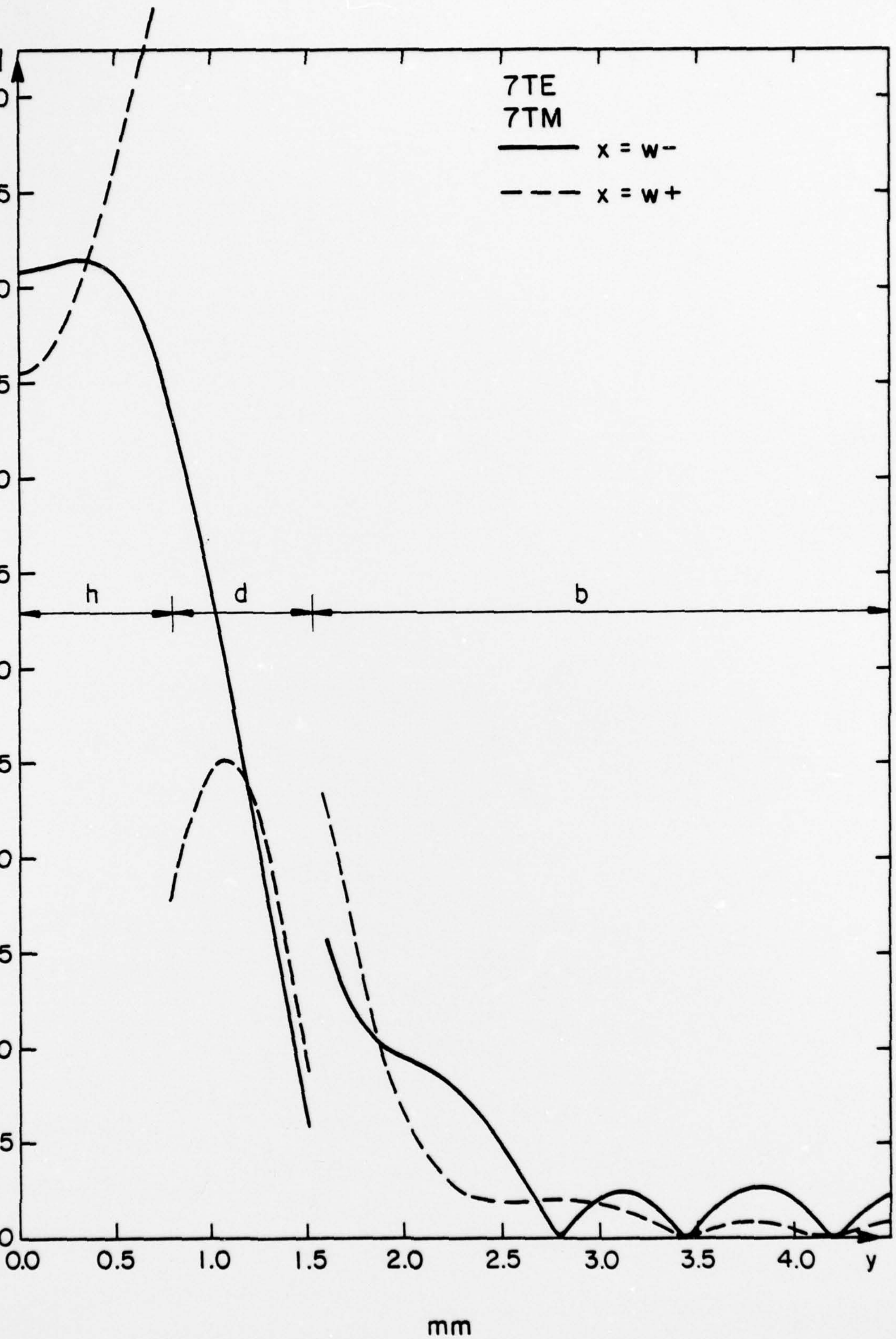

Figure 4. Plot of $|E_y|$ field of the E_{11}^{y} mode at the interface for $x = w^{-}$ and $x = w^{+}$.

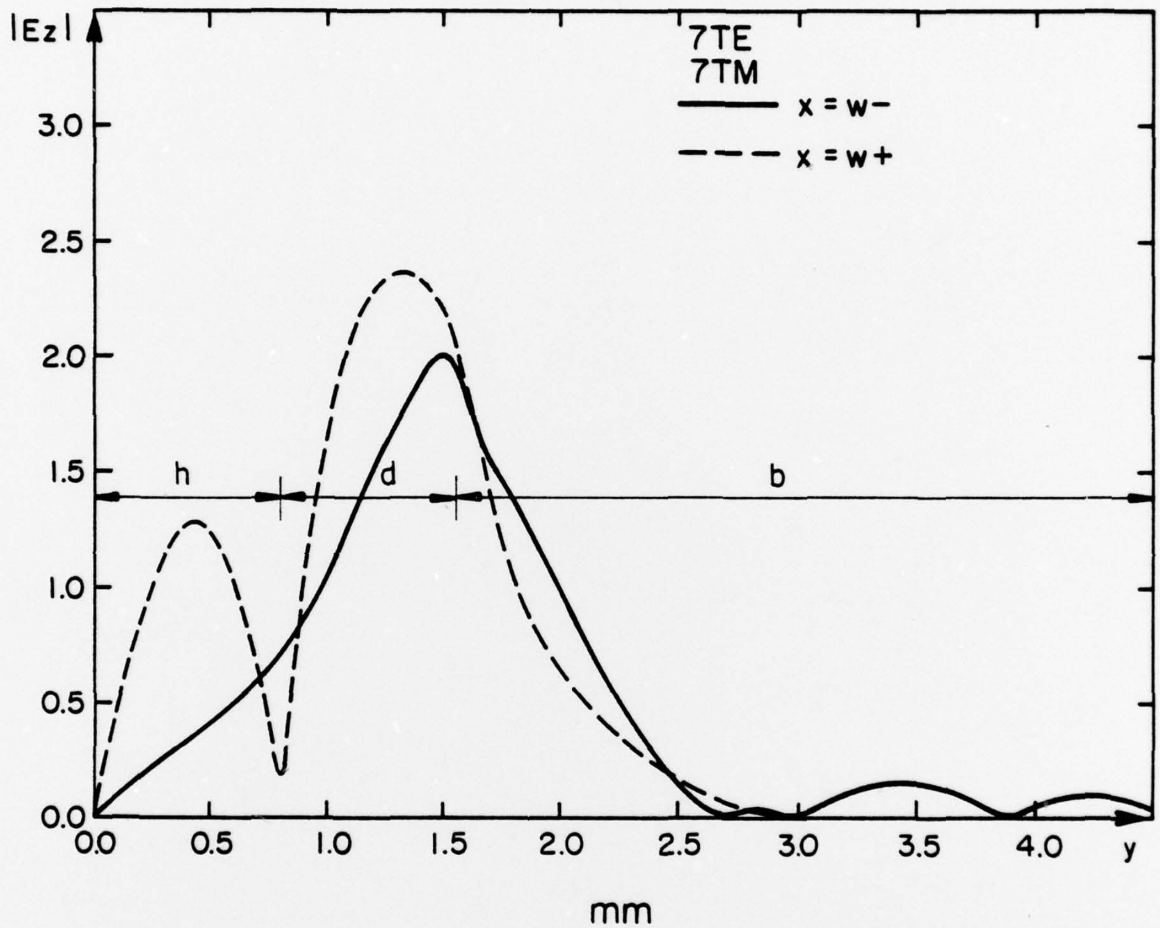

Figure 5. Plot of $|E_z|$ field of the E^y_{11} mode at the interface $x = w^-$ and $x = w^+$.

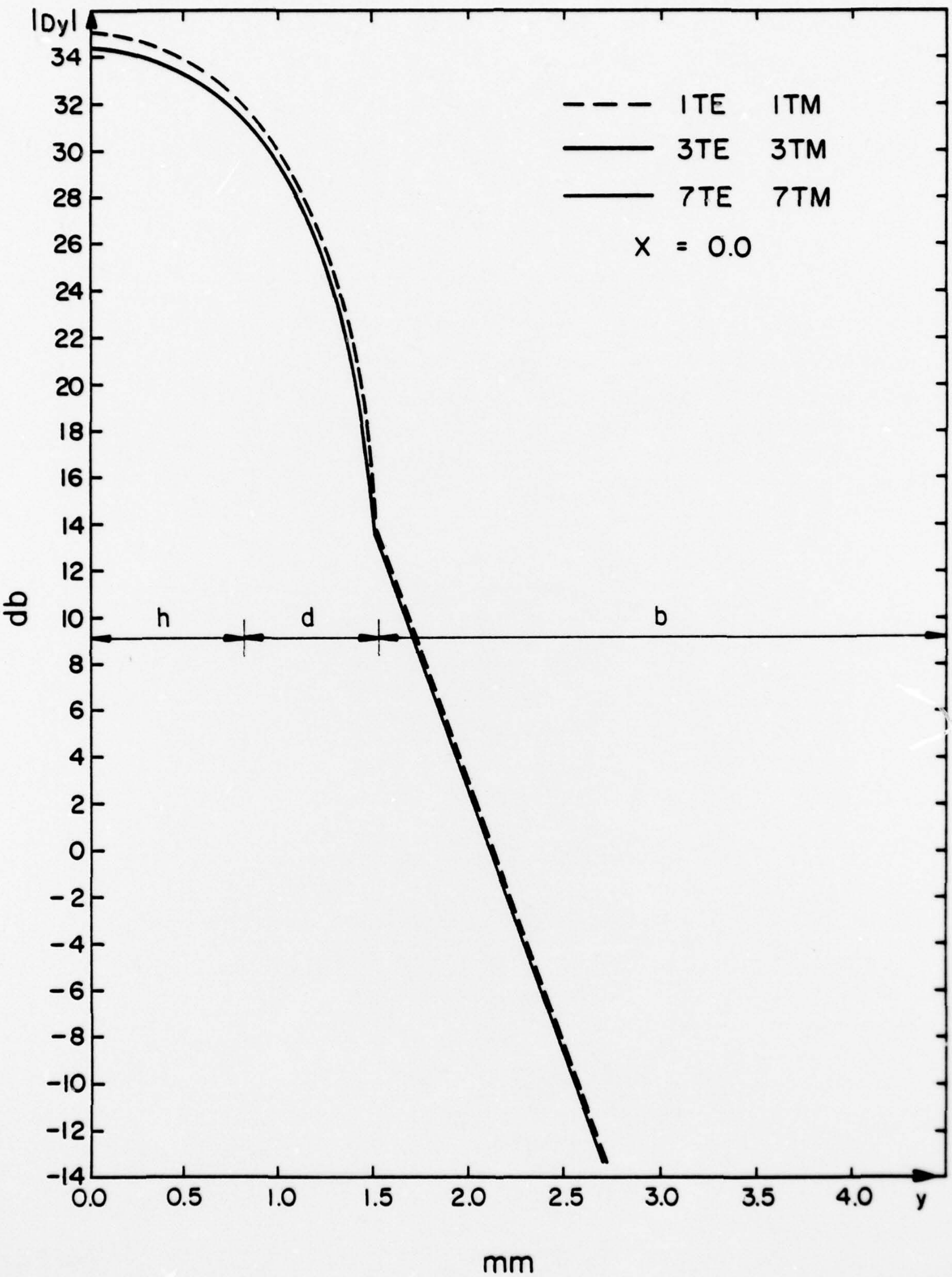

Figure 6. Plot of $|D_y|$ field of the E_{11}^y mode in the vertical direction for x = 0.0.

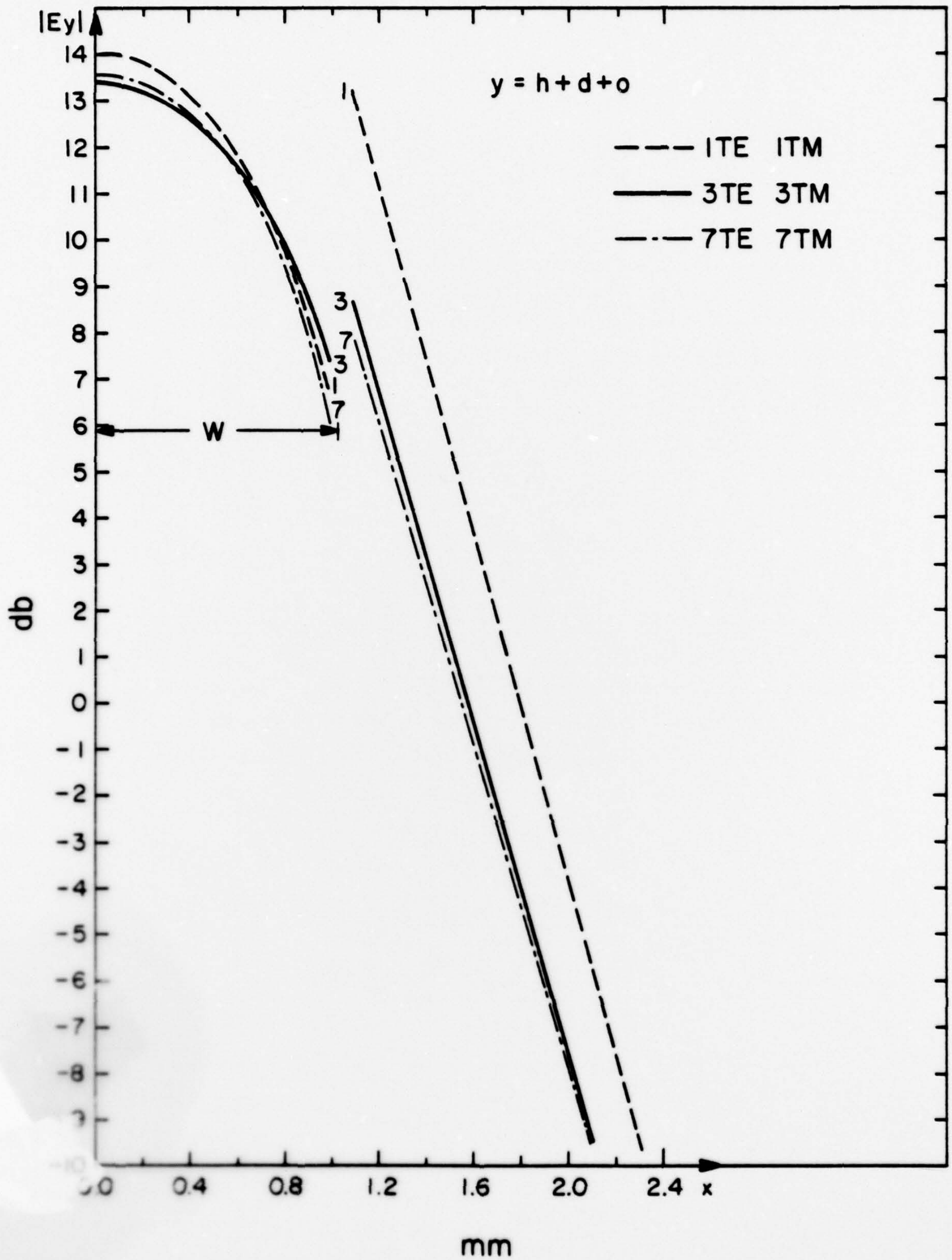

Figure 7. Plot of $|E_y|$ field of the E^y_{11} mode in the transverse direction for $y = h + d + 0$.

plane. The field is seen to be extremely small at the top shield located at $y = h + d + b$, with $b = 2(d+h)$, thus justifying our original assumption that the perturbation introduced by the perfect electric conductor placed at the top of the guide is negligible. Figure 7 shows the distribution of E_y at the $y = h + d + 0^+$ plane. It is evident from this diagram that most of the energy carried by a guided wave is confined within the strip region.

5.2. Results of Variational Calculations

The field distributions calculated via the mode-matching method are used in the modified variational formula for the propagation constant as given by Equations (19) and (20) in Section 4. Some calculated values of k_z, using this mixed-field formulation, are presented in Table II and are compared with the results obtained directly from the solution of the eigenvalue problem in the mode-matching technique.

TABLE II.

PROPAGATION CONSTANTS CALCULATED VIA VARIATIONAL METHOD. COMPARISON IS MADE WITH MODE-MATCHING RESULTS GIVEN IN TABLE I.

A.

E^y_{11}	1TE 1TM	3TE 3TM	5TE 5TM
Mixed field formula (vari.)*	2.98871	2.98588	
Mode Matching	2.97181	2.98916	2.987208

B.

H^y_{11}	1TE 1TM	3TE 3TM	5TE 5TM
Mixed field formula (vari.)*	2.74028	2.61385	
Mode Matching	2.73411	2.62101	2.72033

C.

E^y_{21}	1TE 1TM	3TE 3TM	5TE 5TM
Mixed Field formula (vari.)*	2.39487	2.37895	
Mode Matching	2.36457	2.38713	2.39104

*vari. stands for variational

APPENDIX I

In establishing the variational expression (19) in Section 4, we have employed the reciprocity theorem as embodied in the reaction concept ([8], [10]). The reaction between infinite traveling-wave line sources located in the interior of a waveguide and the fields radiated by them can be defined as follows [10]:

$$\langle a,b\rangle = \iint [\vec{J}_a(x,y)\ \sigma\ \vec{E}_b(x,y) + \vec{K}_a(x,y)\ \sigma\ \vec{H}_b(x,y)]\ dx\ dy \qquad \text{(I-1)}$$

in which

$$\sigma = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix},$$

$$\vec{\mathcal{J}} = \vec{J}\ (x,y)\ e^{-jk_z z}$$

$$\vec{\mathcal{K}} = \vec{K}\ (x,y)\ e^{-jk_z z}$$

$$\vec{\mathcal{E}} = \vec{E}\ (x,y)\ e^{-jk_z z}$$

$$\vec{\mathcal{H}} = \vec{H}\ (x,y)\ e^{-jk_z z}$$

The double integration in (I-1) is performed over the cross section of the guide. In view of the definitions of +z and -z traveling waves (as given by Harrington [8]) in Equations (17) and (18) and similar definitions for the sources as given below

$$
\begin{cases}
\vec{J}^{+} = \vec{J}^{+}(x,y)\, e^{-jk_z z} = (\vec{J}_t + \hat{z}\, J_z)\, e^{-jk_z z} \\
\vec{K}^{+} = \vec{K}^{+}(x,y)\, e^{-jk_z z} = (\vec{K}_t + \hat{z}\, k_z)\, e^{-jk_z z}
\end{cases}
\tag{I-2}
$$

$$
\begin{cases}
\overleftarrow{J}^{-} = \overleftarrow{J}^{-}(x,y)\, e^{+jk_z z} = (\vec{J}_t - \hat{z}\, J_z)\, e^{+jk_z z} \\
\overleftarrow{K}^{-} = \overleftarrow{K}^{-}(x,y)\, e^{+jk_z z} = (-\vec{K}_t + \hat{z}\, K_z)\, e^{+jk_z z}\, .
\end{cases}
\tag{I-3}
$$

Ramsey's reaction formula (I-1) for the uniform waveguides can be written in the following equivalent forms:

$$
\begin{aligned}
\langle a,b\rangle &= \iint (\vec{J}_a^{+} \cdot \overleftarrow{E}_b^{-} - \vec{K}_a^{+} \cdot \overleftarrow{H}_b^{-})\, dx\, dy \\
&= \iint (\overleftarrow{J}_a^{-} \cdot \vec{E}_b^{+} - \overleftarrow{K}_a^{-} \cdot \vec{H}_b)\, dx\, dy \\
&= \iint (\vec{J}_a^{+} \cdot \overleftarrow{E}_b^{-} - \overleftarrow{K}_a^{-} \cdot \vec{H}_b^{+})\, dx\, dy \\
&= \iint (\overleftarrow{J}_a^{-} \cdot \vec{E}_b^{+} - \vec{K}_a^{+} \cdot \overleftarrow{H}_b^{-})\, dx\, dy.
\end{aligned}
\tag{I-4}
$$

It can be shown [8] that

$$\langle a, a \rangle = 0 \tag{I-5}$$

is a stationary formula for a trial field "a" and the associated source. For the waveguide problem, the stationary formula for the propagation constant is found from (I-5) using the definition of the reaction as given in (I-1) or any of the equivalent forms given in (I-4). For instance, Harrington [8], in effect, uses the fourth expression for the reaction in (I-4) to obtain the variational expression for the propagation constant.

In this paper, we use the same expression for the reaction (fourth in (I-4)) in the formulation of the variational expression. Starting from Maxwell's equations, the electric and magnetic sources are found to be

$$\begin{cases} -\vec{K}^{+} = \nabla \times \vec{E}^{+} + j\omega\mu\vec{H}^{+} - jk_z\hat{z} \times \vec{E}^{+} \\ \\ \vec{J}^{+} = \nabla \times \vec{H}^{+} - j\omega\mu\vec{E}^{+} - jk_z\hat{z} \times \vec{H}^{+} \end{cases} \tag{I-6}$$

and

$$\begin{cases} -\vec{K}^{-} = \nabla \times \vec{E}^{-} + j\omega\mu\vec{H}^{-} + jk_z\hat{z} \times \vec{E}^{-} \\ \\ \vec{J}^{-} = \nabla \times \vec{H}^{-} - j\omega\mu\vec{E}^{-} + jk_z\hat{z} \times \vec{H}^{-} \end{cases} \tag{I-7}$$

These sources support the fields and can be used in (I-5). However, if the fields are discontinuous along a certain interface "c" in the cross section of the guide, additional surface currents, as given below, are needed to support the discontinuity.

$$\left\{\begin{aligned} -\vec{K}_s^{+} &= \vec{n} \times (\vec{E}_p^{+} - \vec{E}_m^{+}) \\ \vec{J}_s^{+} &= \hat{n} \times (\vec{H}_p^{+} - \vec{H}_m^{+}) \end{aligned}\right. \tag{I-8}$$

and

$$\left\{\begin{aligned} -\vec{K}_s^{-} &= \hat{n} \times (\vec{E}_p^{-} - \vec{E}_m^{-}) \\ \vec{J}_s^{-} &= \hat{n} \times (\vec{H}_p^{-} - \vec{H}_m^{-}) \end{aligned}\right. \tag{I-9}$$

in which subscripts "p" and "m" refer to the opposite sides of the surface of discontinuity and $\hat{n}$ is a unit normal directed from "m" to "p."

The variational formula (I-5) using the fourth expression of the reaction in (I-4) is written as

$$\begin{aligned} \langle a, a \rangle = &\iint_s (\vec{J}^{-} \cdot \vec{E}^{+} - \vec{K}^{+} \cdot \vec{H}^{-}) \, ds \\ &+ \int_c (\vec{J}_s^{-} \cdot \vec{E}^{+} - \vec{K}_s^{+} \cdot \vec{H}^{-}) \, d\ell = 0 \end{aligned} \tag{I-10}$$

in which double integration is over the surface of the cross-section of the guide and line integration is along "c," the line of field discontinuity in the cross-section.

The integrand in the line integral of (I-10) is not, however, well-defined, due to the fact that either $\vec{E}_m^{+}$ or $\vec{E}_p^{+}$ and also either $\overleftarrow{H}_m^{-}$ or $\overleftarrow{H}_p^{-}$ can be used for $\vec{E}^{+}$ and $\overleftarrow{H}^{-}$ in the integrand. This is, however, clarified by Ramsey [10], in the sense that either "m" or "p" values should be used for both $\vec{E}$ and $\vec{H}$ and, in either case, the same result is obtained.

Taking $\vec{E}^{+} = \vec{E}_p^{+}$ and $\vec{H}^{+} = \vec{H}_p^{+}$ and substituting currents as given in (I-6), (I-7), (I-8) and (I-9) in (I-10) and effecting some rearrangements, we end up with the variational expression (19).

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